Problem Solving Strategies 1

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$$S_n = 1+2+3+\cdots+n$$
$$S_n = n+(n-1)+(n-2)+\cdots+1$$
$$2S_n = (n+1)+(n+1)+(n+1)+\cdots+(n+1)$$
$$\Rightarrow 2S_n = n(n+1)$$
$$\Rightarrow S_n = \frac{n(n+1)}{2} \rightarrow \text{sum of 1st } n \text{ natural numbers}$$

$$O_n = 1+3+5+\cdots+2n+1$$
$$O_n = S_{2n+1} - E_n$$
$$= \frac{(2n+1)(2n+2)}{2} - n(n+1)$$
$$= (2n+1)(n+1) - n(n+1)$$
$$= (n+1)(2n+1-n) = (n+1)^2$$

$$E_n = 2+4+6+\cdots+2n$$
$$\Rightarrow E_n = 2(1+2+\cdots+n)$$
$$\Rightarrow E_n = \frac{n(n+1)}{2}2 = n(n+1)$$

$$\cancel{1^2} - 0^2 = 1$$
$$\cancel{2^2} - \cancel{1^2} = 3$$
$$\cancel{3^2} - \cancel{2^2} = 5$$
$$\vdots$$
$$n^2 - \cancel{(n-1)^2} = 2n-1$$
$$n^2 = 1+3+\cdots+2n-1 = \sum_{k=1}^{n}(2k-1)$$

$$\Rightarrow \sum_{k=1}^{n}(2k-1) = n^2$$
$$\Rightarrow 2\sum_{k=1}^{n}k - \sum_{k=1}^{n}1 = n^2$$
$$\Rightarrow 2\sum_{k=1}^{n}k - n = n^2$$
$$\Rightarrow \sum_{k=1}^{n}k = \frac{n^2+n}{2} = \frac{n(n+1)}{2}$$

$$S = 1^2+2^2+\cdots+n^2$$

$$n^3 - (n^3 - 3n^2 + 3n - 1)$$
$$= 3n^2 - 3n + 1$$

$S = 1^2 + 2^2 + \cdots + n^2$

$n^3 - (n^3 - 3n^2 + 3n - 1)$

$= 3n^2 - 3n + 1$

$\cancel{1^3} - 0^3 = 1$

$\cancel{2^3} - \cancel{1^3} = 7$

$\vdots$

$n^3 - \cancel{(n-1)^3} = 3n^2 - 3n + 1$

$n^3 = \sum_{k=1}^{n} (3k^2 - 3k + 1)$

$\Rightarrow n^3 = 3\sum_{k=1}^{n} k^2 - 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 = 3\sum_{k=1}^{n} k^2 - 3\frac{n(n+1)}{2} + n$

$\Rightarrow 3\sum_{k=1}^{n} k^2 = n^3 + 3\frac{n(n+1)}{2} - n$

$\Rightarrow \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$S = 1^3 + 2^3 + \cdots + n^3$ same process

\# So using the same process we proved that we can find the value $1^m + 2^m + \cdots + n^m$. as after the telescopic sum every term will have power less than m

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$S = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{99\cdot 100} = \sum_{k=1}^{99} \frac{1}{k(k+1)}$

$S = \sum_{k=1}^{99} \left(\frac{(k+1) - k}{k(k+1)} \right) = \sum_{k=1}^{99} \left(\frac{1}{k} - \frac{1}{k+1} \right) = \frac{1}{1} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cdots + \cancel{\frac{1}{99}} - \frac{1}{100}$

$= 1 - \frac{1}{100} = \frac{99}{100}$

$S = \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

• x, y, z are ages of three persons, $xyz = 36$. $x+y+z = w$

knowing this information doesn't give you their ages

> x, y, z are ages of three ...
and knowing this information doesn't give you their ages for some w.

$$36 = 2^2 3^2 = 2 \times 2 \times 3 \times 3$$

$(x,y,z) = (1,1,36),\ (1,2,18),\ (1,3,12),\ (1,4,9),\ (1,6,6),\ (2,2,9),\ (2,3,6),$

$x+y+z = \quad 38,\quad 21,\quad 16,\quad 14,\quad \underbrace{13,\quad 13},\quad 11,$

$(3,3,4)$

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So $w = 13$.

Q> $A = \{a_1, a_2, \ldots, a_{20}\}$ and a_i's are all distinct.

$a_i \in \{1, 4, 7, \ldots, 100\}$ this AP

Prove that there is two distinct integers in A whose sum is 104.

$100 = 1 + 33 \times 3 \rightarrow$ So there are 34 elements in the AP

Ans:— $d = 3$,

18 rows

$\rightarrow$ $1 \rightarrow$ not needed

$\rightarrow$ $4 + 100 = 104$

$\rightarrow$ $7 + 97 = 104$

$\vdots$

$\rightarrow$ $48 + 55 = 104$

$\rightarrow$ $52 \rightarrow$ not needed

$\frac{32}{2} = 16$ combinations

1, 52, and one number from each of 16 combinations is a valid subset of 18 elements

But if I take one more in any way we get a combination of the 16 combinations. So $\exists$ a pair such that there sum is 104